The final exam answer key. (**Phys-atom&nucl**)
2nd year SM Physics 2024-2025

Exercise 1 (8 pts):

1- Mass excess: $\delta(A,Z) = \mathcal{M} - A \rightarrow \mathcal{M} = \delta + A$

2. $$\begin{aligned} M &= \mathcal{M} - Zm_e = \delta(A,Z) + A - Zm_e \\ B &= (Zm_p + Nm_n - M)c^2 ......A = Z + N \\ &= (Zm_p + Nm_n - (\delta(A,Z) + A - Zm_e))c^2 \\ &= (Z(m_p + m_e) + Nm_n - \delta(A,Z) - A)c^2 \\ &= (Z(m_p + m_e) + Nm_n - \delta(A,Z) - Z - N)c^2 \\ &= (Z(m_p + m_e - 1) + N(m_n - 1) - \delta(A,Z))c^2 \\ &= (Z(m_H - 1) + N(m_n - 1) - \delta(A,Z))c^2 \\ &= [Z\delta(1,1) + N\delta(1,1) - \delta(A,Z)]c^2 \\ &= (Z\delta_H + N\delta_n - \delta_X)c^2 \end{aligned}$$

where $\delta_H = \delta(1,1) = m_H - 1 = m_p + m_e - 1$ is the excess mass of the hydrogen atom

and $\delta_n = \delta(1,0) = m_n - 1$ the excess mass of the neutron.

- Neutron separation: ${}^A_Z X + S_n \rightarrow {}^{A-1}_{Z} Y + {}^1_0 n$

$$\begin{aligned} S_n &= (M(A\text{-}1,Z) + m_n - M(A,Z))c^2 \\ & [(\delta(A\text{-}1,Z) + (A\text{-}1) - Zm_e + m_n \\ & -(\delta(A,Z) + A - Zm_e)]c^2 \\ & [\delta(A\text{-}1,Z) - \delta(A,Z) + m_n - 1]c^2 \\ &= [\delta(A\text{-}1,Z) - \delta(A,Z) + \delta_n]c^2 \\ &= [\delta_Y - \delta_X + \delta_n]c^2 \end{aligned}$$

- Proton separation: ${}^{A-1}_{Z} X + S_p \rightarrow {}^{A-1}_{Z-1} Y + {}^1_1 p$

$$\begin{aligned} S_p &= (M(A\text{-}1,Z\text{-}1) + m_p - M(A,Z))c^2 \\ &= [\delta(A\text{-}1,Z\text{-}1) + (A\text{-}1) - (Z\text{-}1)m_e + m_p \\ &= -(\delta(A,Z) + A - Zm_e)]c^2 \\ &= [\delta(A\text{-}1,Z\text{-}1) - 1 + m_e + m_p - \delta(A,Z))]c^2 \\ &= [\delta(A\text{-}1,Z\text{-}1) + \delta_H - \delta(A,Z))]c^2 \\ S_p &= [\delta_Y - \delta_X + \delta_H)]c^2 \end{aligned}$$

3- Numerical Application : for ${}^{105}_{46}\text{X} \equiv {}^{105}_{46}\text{Pd}$:

$$\begin{aligned} \mathcal{M} &= \delta(105,46) + 105 = 104.905\,08u \\ &= -0.094918 + 105 = 104.905\,08u \\ M &= \delta(105,46) + 105 - 46 \times 5.4858 \times 10^{-4} = 104.879\,85u \\ B &= (Z(m_p + m_e - 1) + N(m_n - 1) - \delta(A,Z))c^2 \\ &= 46(1.007\,277 + 5.4858 \times 10^{-4} - 1) \\ & +59(1.008665 - 1) - (-0.094918) \\ &= 0.966103 \times 931.5 = 899.925 \text{ MeV} \end{aligned}$$

Neutron separation: ${}^{105}_{46}\text{X} + S_n \rightarrow {}^{104}_{46}\text{Y} + {}^1_0 n$

$$\begin{aligned} S_n &= [\delta_Y - \delta_X + \delta_n)]c^2 = [\delta_Y - \delta_X + m_n - 1)]c^2 \\ &= \delta(104,46) - \delta(105,46) + 1.008665 - 1 \\ &= -0.095969 - (-0.094918) + 1.008665 - 1 \\ &= 0.007614 \times 931.5 = 7.092 \text{ MeV} \end{aligned}$$

Proton separation: ${}^{105}_{46}\text{X} + S_p \rightarrow {}^{104}_{45}\text{Y} + {}^1_0 p$

$$\begin{aligned} S_p &= [\delta_Y - \delta_X + \delta_H)]c^2 = [\delta_Y - \delta_X + m_p + m_e - 1)]c^2 \\ &= [\delta(104,45) - \delta(105,46) + m_p + m_e - 1]c^2 \\ &= -0.093348 + 0.094918 + 1.007\,277 + 5.4858 \times 10^{-4} - 1 \\ &= 0.009\,396 \times 931.5 = 8.752 \text{ MeV} \end{aligned}$$

Exercise 2 (7 pts):

1) The α decay eq. : ${}^A_Z\text{X} \rightarrow {}^{A-4}_{Z-2}Y + {}^{A-4}_{Z-2}\alpha$

$B(A,Z)$ can be approximated (for $A \gg 1$) by:

$$B(A,Z) = a.A + b.A^2 = 9.402 \times A - 7.7 \times 10^{-3} \times A^2$$

2) Energy conservation:

$\text{M}(X)\,c^2 + E_c^X = \text{M}(Y)\,c^2 + \text{M}(\alpha)\,c^2 + E_c^\alpha + E_c^Y$

$(Zm_p + Nm_n)\,c^2 - B_X + E_c^X = ((Z\text{-}2)\,m_p + (N\text{-}2)\,m_n)\,c^2 - B_Y + (2m_p + 2m_n)\,c^2 - B_\alpha + E_c^\alpha + E_c^Y$

$\rightarrow -B_X + E_c^X = -B_Y - B_\alpha + E_c^\alpha + E_c^Y$

In a reference frame attached to the nuclide ${}^A_Z\text{X}$ ($E_c^X = 0$).

$\rightarrow B_Y - B_X + B_\alpha = E_c^\alpha + E_c^Y = Q$

For the disintegration to be spontaneous, Q must be positive.: $Q > 0$

$\rightarrow Q = B_Y - B_X + B_\alpha > 0$

3) $Q = -aA + bA^2 + a\,(A-4) - b\,(A-4)^2 + B\,(\alpha) > 0$

$\rightarrow 8Ab - 16b - 4a + B\,(\alpha) > 0$

$\rightarrow A > 2 + a/\,(2b) - B\,(\alpha)\,/\,(8b) > 0$

4) $A > 2 + \frac{9.402}{2 \times 7.7 \times 10^{-3}} - \frac{28.3}{8 \times 7.7 \times 10^{-3}} = 153.\,103\,9$

So for α decay to be energetically possible it is necessary that $A > 153$.

Exercise 3 (5 pts):

The radioactive decay laws for the two elements A et B:

$N_A = N_A^0 e^{-\lambda_A t}$ (*) $\qquad N_B = N_B^0 e^{-\lambda_B t}$ (**)

As the alloy is initially composed of **equal parts in mass** of the two metals A and B:

$m_0^A = m_0^B \rightarrow N_A^0 = N_B^0$

because $N = \frac{m}{M} N_{Avog}$ (M : molar mass. N_{Avog} : Avogadro number)

1. $\frac{(*)}{(**)} \rightarrow \frac{N_A}{N_B} = e^{(\lambda_B - \lambda_A)t}$

$$\rightarrow t = \frac{\ln\left(\frac{N_A}{N_B}\right)}{\lambda_B - \lambda_A} = \frac{1}{\ln 2} \frac{\ln\left(\frac{N_A}{N_B}\right)}{\frac{1}{t^B_{1/2}} - \frac{1}{t^A_{1/2}}} = \frac{1}{\ln 2} \frac{\ln\left(\frac{\frac{m_A}{M_A} N_{Avog}}{\frac{m_B}{M_B} N_{Avog}}\right)}{\frac{1}{t^B_{1/2}} - \frac{1}{t^A_{1/2}}}$$

where $N_A = \frac{m_A}{M_A} N_{Avog}$ and $N_B = \frac{m_B}{M_B} N_{Avog}$

$M_A = M_B$ (*the molar masses are assumed to be equal*).

$$t = \frac{1}{\ln 2} \frac{\ln\left(\frac{m_A}{m_B}\right)}{\frac{1}{t^B_{1/2}} - \frac{1}{t^A_{1/2}}} = \frac{1}{\ln 2} \frac{\ln\left(\frac{2.2}{0.55}\right)}{\frac{1}{12} - \frac{1}{18}} = 72.0 \textit{ years}$$

So the age of the alloy is 72.0 *years*