

## Exam: Mathematics 4

### Exercise 1 (04 pts).

*Solve the following equations in  $\mathbb{C}$ :*

1.  $z^6 + 1 = i\sqrt{3}$ ,
2.  $z^3 - 3z^2 + 3z + 1 = 0$ .

### Exercise 2 (06 pts).

1. *Show that the function  $u$  defined below is harmonic.*

$$u(x, y) = y \cos y \operatorname{ch} x + x \sin y \operatorname{sh} x, \quad x, y \in \mathbb{R}.$$

2. *Find a function  $v$  so that the function  $f = u + iv$  is holomorphic.*

### Exercise 3 (07 pts).

*Calculate  $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ . the length of:*

- a) *The parabola  $x = 2t$ ,  $y = t^2 + 3$ ,*
- b) *The broken line formed by the line segments  $(0, 3) \rightarrow (2, 3)$  and  $(2, 3) \rightarrow (2, 4)$ ,*
- c) *The line segment with endpoints  $(0, 3)$  and  $(2, 4)$ .*

### Exercise 4 (03 pts).

1. *Expand in Laurent series around its singular point  $f(z) = \frac{1}{z^2}e^{\frac{1}{z}}$ .*
2. *Specify the nature of its singular point.*
3. *Deduce the residue of  $f(z)$  at this singular point.*

Good luck