

Exercise N° 01: (6pts)

Three identical point charges: $q_A = q_B = q_C = +4\mu C$, are placed respectively on three points on (Oxy) plan; $A(-2, 0)$; $B(0, -1)$ and $C(3, 0)$, with length unit is the centimeter.

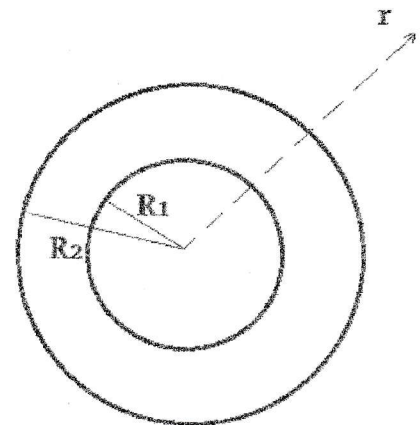
- 1- Calculate and represent the electric field vector at the origin O.
- 2- Calculate the electric potential in the origin.

We give: $K = \frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 Nm^2/C^2$

Exercise N° 02: (6pts)

Consider two charges $Q_1 = -Q$ and $Q_2 = 3Q$ ($Q > 0$) distributed uniformly on surface of two concentric spheres with radius are $R_1 = R$ and $R_2 = 2R$.

- 1- Give the relationships between the charge densities (σ_1, σ_2) and Q and R .
- 2- Using Gauss's law; calculate the electric field as a function of (r, Q , and ϵ_0) at a distance " r " from the center of the spheres.
- 3- Knowing that $V(R_1) = -V_0$ and $V(R_2) = V_0$, calculate the electric potential at the distance " r ".



Exercise N° 03: (3pts)

Consider a parallel plate capacitor formed by two conductor plates of area $A = 2cm$ separated by a dielectric of permittivity $\epsilon = 5 \cdot 10^{-9} F/cm$ and thickness of $20 \mu m$.

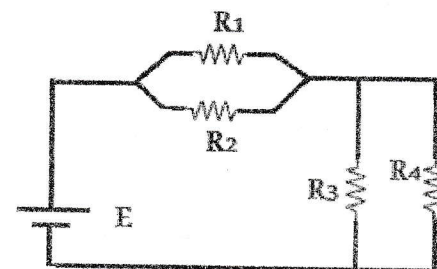
- 1- Calculate its capacitance.
- 2- If we make this capacitor into a cylindrical shape of radius $R_1 = 3,18 mm$ and $R_2 = 3,20 mm$ and height $h = 1cm$, what will its capacitance?
- 3- Compare the two values.

Exercise N° 04: (5pts)

Consider the opposite circuit.

- 1- Calculate the equivalent resistance.
- 2- Using current divisor; calculate and represent the currents flowing through each branch.

Numerical application: $E = 10V$, $R_1 = 10\Omega$, $R_2 = 15\Omega$,
 $R_3 = 15\Omega$, $R_4 = 30\Omega$.



Exercise N° 01 (06 Pts)

$$\vec{E}_O = \vec{E}_{AO} + \vec{E}_{BO} + \vec{E}_{CO}$$

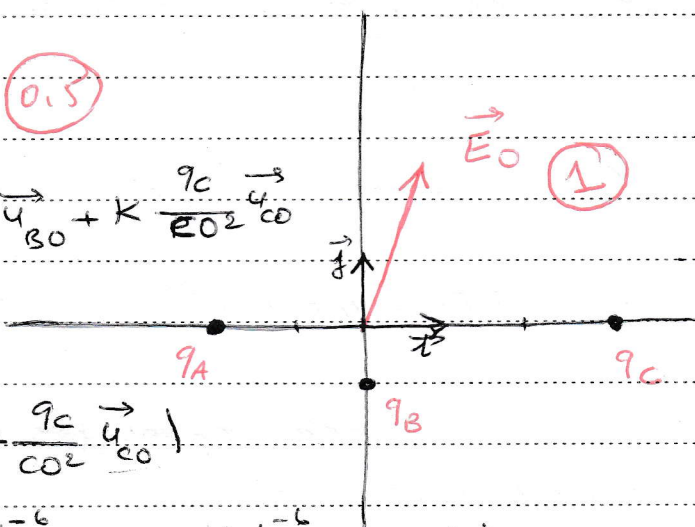
0.5

$$\vec{E}_O = K \frac{q_A}{AO^2} \vec{u}_{AO} + K \frac{q_B}{BO^2} \vec{u}_{BO} + K \frac{q_C}{CO^2} \vec{u}_{CO}$$

1

 \vec{E}_O

1



$$\vec{E}_O = K \left(\frac{q_A}{AO^2} \vec{u}_{AO} + \frac{q_B}{BO^2} \vec{u}_{BO} + \frac{q_C}{CO^2} \vec{u}_{CO} \right)$$

$$\vec{E}_O = 9 \cdot 10^9 \left(\frac{4 \cdot 10^{-6}}{(2 \cdot 10^{-2})^2} \vec{i} + \frac{4 \cdot 10^{-6}}{(1 \cdot 10^{-2})^2} \vec{j} + \frac{4 \cdot 10^{-6}}{(3 \cdot 10^{-2})^2} (-\vec{i}) \right)$$

$$\vec{E}_O = 9 \cdot 10^9 \cdot 4 \cdot 10^{-6} \cdot 10^4 \left(\frac{1}{4} \vec{i} - \frac{1}{9} \vec{i} + \vec{j} \right)$$

$$\vec{E}_O = 36 \cdot 10^7 (0,14 \vec{i} + \vec{j})$$

or

$$\vec{E}_O = 5 \cdot 10^7 \vec{i} + 36 \cdot 10^7 \vec{j}$$

1

$$V_O = V_{AO} + V_{BO} + V_{CO}$$

0.5

$$V_O = K \frac{q_A}{AO} + K \frac{q_B}{BO} + K \frac{q_C}{CO}$$

1

$$V_O = 9 \cdot 10^9 \left(\frac{4 \cdot 10^{-6}}{2 \cdot 10^{-2}} + \frac{4 \cdot 10^{-6}}{1 \cdot 10^{-2}} + \frac{4 \cdot 10^{-6}}{3 \cdot 10^{-2}} \right)$$

$$V_O = 66 \cdot 10^5 \text{ V}$$

1

Exercise N° 02:

①

We have $Q_1 = \sigma_1 \cdot S_1 = \sigma_1 \cdot 4\pi R_1^2$

and $Q_2 = \sigma_2 \cdot S_2 = \sigma_2 \cdot 4\pi R_2^2$

$$Q_1 = -Q = 4\pi R_1^2 \sigma_1 \Rightarrow \sigma_1 = \frac{-Q}{4\pi R_1^2}$$

$$Q_2 = 3Q = 4\pi (2R)^2 \sigma_2 \Rightarrow \sigma_2 = \frac{3Q}{16\pi R^2}$$

② We have Three cases for ($r < R_1$; $R_1 < r < R_2$; $r > R_2$)

Gauss's law

$$\oint \vec{E} \cdot d\vec{S} = \frac{\sum Q_{\text{ins}}}{\epsilon_0}$$

For $r < R_1$

$$\oint \vec{E} \cdot d\vec{S} = E \cdot S_c = E \cdot (4\pi r^2)$$

$$Q_{\text{ins}} = 0 \Rightarrow E = 0$$

$$V = - \int \vec{E} \cdot d\vec{r} = C_1$$

$$V = \text{constant}$$

For $R_1 < r < R_2$

$$Q_{\text{ins}} = Q_1 = -Q$$

$$E \cdot 4\pi r^2 = \frac{-Q}{\epsilon_0} \Rightarrow E = \frac{-Q}{4\pi \epsilon_0 r^2}$$

$$V = - \int \vec{E} \cdot d\vec{r} = - \int \frac{-Q}{4\pi \epsilon_0} \frac{1}{r^2} dr$$

$$V = - \frac{Q}{4\pi \epsilon_0} \frac{1}{r} + C_2$$

For $r > 2R$

$$Q_{\text{ins}} = 3Q - Q = 2Q$$

$$E \cdot 4\pi r^2 = \frac{2Q}{\epsilon} \Rightarrow E = \frac{2Q}{4\pi\epsilon_0 r^2}$$

$$V = \frac{2Q}{4\pi\epsilon_0 r} + C_3$$

Exercise N° 03

For a parallel plate capacitor
we have

$$C = \frac{\epsilon S}{d} = \frac{\epsilon A}{d}$$

$$C = 5 \cdot 10^{-9} \frac{21}{20 \cdot 10^{-4}} = 0,5 \cdot 10^{-5} \text{ F} = 5 \cdot 10^{-6} \text{ F}$$

$$C = 5 \mu\text{F}$$

For a cylindrical capacitor we have

$$C = 2\pi\epsilon \frac{h}{\ln \frac{R_2}{R_1}}$$

$$C = 2,3,14 \cdot 5 \cdot 10^{-9} \frac{1}{\ln \frac{3,20}{3,18}} = 5,0082 \cdot 10^{-6} \text{ F}$$

$$C(\text{Plate}) \simeq C(\text{Cylinder})$$

or $C(\text{Cylinder}) > C(\text{Plate})$

Exercise N° 04

①

$$R_{eq} = (R_1 // R_2) + (R_3 // R_4)$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 \cdot R_4}{R_3 + R_4}$$

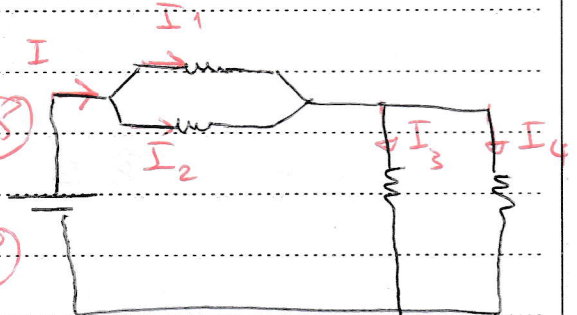
$$R_{eq} = 16 \, \Omega \quad (1)$$

$$I_{tot} = \frac{E}{R_{eq}} = \frac{10}{16} = 0,625 \, A \quad (1)$$

0,5

$$I_1 = \frac{R_2}{R_1 + R_2} I = 0,375 \, A \quad (0,5)$$

$$I_2 = \frac{R_1}{R_1 + R_2} I = 0,250 \, A \quad (0,5)$$



0,5

$$I_3 = \frac{R_4}{R_3 + R_4} I = 0,416 \, A \quad (0,5)$$

$$I_4 = \frac{R_3}{R_3 + R_4} I = 0,208 \, A \quad (0,5)$$