Exam Solution

Phys 2

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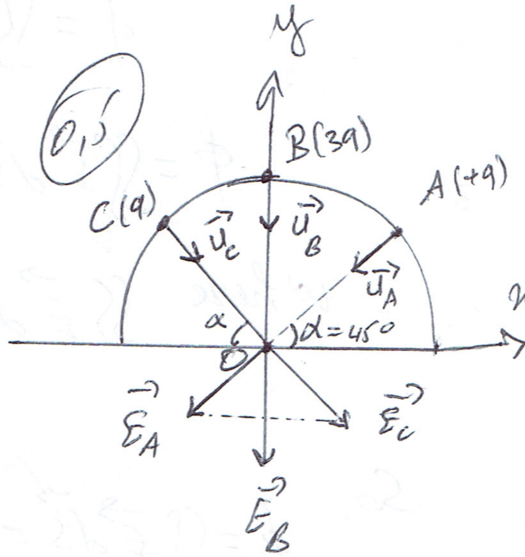
Exo1 (06 points)

1) The electric field at the point O is

$$\vec{E}_0 = \vec{E}_B + \vec{E}_A + \vec{E}_c \qquad 0{,}5$$



$$0{,}5 \quad \begin{cases} \vec{E}_A = kq \dfrac{1}{R_A^2}\vec{U}_A \\ \vec{E}_B = k\,3q \dfrac{1}{R_B^2}\vec{U}_B \\ \vec{E}_C = kq \dfrac{1}{R_C^2}\vec{U}_C \end{cases} \quad , k = \frac{1}{4\pi\varepsilon_0}$$

With
$$0{,}5 \quad \begin{cases} R_A = R_B = R_C = R \\ \vec{U}_A = -\cos\alpha\,\vec{i} - \sin\alpha\,\vec{j} \\ \vec{U}_B = -\vec{j} \\ \vec{U}_C = \cos\alpha\,\vec{i} - \sin\alpha\,\vec{j} \end{cases}$$

We have $\cos\alpha = \sin\alpha = \frac{\sqrt{2}}{2}$

$$0{,}5 \quad \begin{cases} \vec{U}_A = -\frac{\sqrt{2}}{2}(\vec{i} + \vec{j}) \\ \vec{U}_B = -\vec{j} \\ \vec{U}_C = \frac{\sqrt{2}}{2}(\vec{i} - \vec{j}) \end{cases} \Rightarrow \begin{cases} \vec{E}_A = -kq \frac{1}{R^2}\frac{\sqrt{2}}{2}(\vec{i} + \vec{j}) \\ \vec{E}_B = -k3q \frac{1}{R^2}\vec{j} \\ \vec{E}_c = kq \frac{1}{R^2}\frac{\sqrt{2}}{2}(\vec{i} - \vec{j}) \end{cases}$$

Therefore, $\vec{E}_0 = \vec{E}_B + \vec{E}_A + \vec{E}_c = -kq\dfrac{(\sqrt{2}+3)}{R^2}\vec{j}$ (1)

2) The ~~the~~ electrostatic force at the point O,

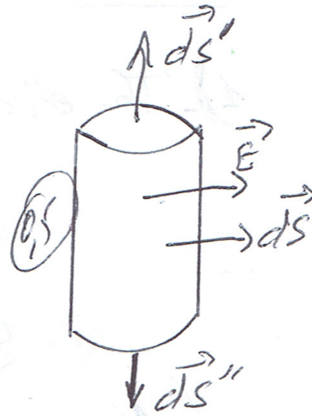
(1) $$\vec{F}_0 = q'\vec{E}_0 = -q\vec{E}_0 = kq^2\frac{(\sqrt{2}+3)}{R^2}\vec{j}$$

3) The potential at the point O,

$$V_0 = V_A + V_B + V_C = kq\frac{(1+1+3)}{R^2} = 5kq\frac{1}{R}$$

Ex 02

1) to calculate the electric field at any point of space, we use the Gauss theorem. The Gauss surface is the cylinder of radius r and height h. (0,25)



The flux across the Gauss surface is:

$$\phi = \iint \vec{E}\,\vec{ds} = \frac{\sum q_{int}}{\varepsilon_0}$$ (0,5)

$$\phi = \iint \vec{E}\,\vec{ds} = \iint E\,ds$$ (0,25)

where $\iint \vec{E}\,\vec{ds}' = \iint \vec{E}\,\vec{ds}'' = 0$ because $\begin{cases} \vec{E} \perp \vec{ds}' \\ \vec{E} \perp \vec{ds}'' \end{cases}$ (0,5)

So, $\phi = \iint \vec{E}\,\vec{ds} = \iint E\,ds = E \iint ds = E\, 2\pi r h$ (0,25)

on other hand

$$\phi = \iint \vec{E}\,\vec{ds} = E\, 2\pi r h = \frac{\sum q_{int}}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\sum q_{int}}{2\pi r h\, \varepsilon_0}$$ (0,5)

1) For $r < R_1$

$q_{int} = 0$ (0,5) $\Rightarrow E_1 = 0$ (0,5)

2) For $R_1 \leq r < R_2$

$dq = \sigma\, ds \Rightarrow \int dq = \sigma \int ds$ (0,25)

$\Rightarrow q_{int} = \sigma\, 2\pi R_1 h$ (0,25)

$\Rightarrow E_2\, 2\pi r h = \sigma\, 2\pi R_1 h$

$\Rightarrow E_2 = \dfrac{\sigma R_1}{\varepsilon_0 r}$ (0,5)

3) For $r \geqslant R_2$

$Q_{int} = Q_1 + Q_2$ with $Q_1 = \sigma\, 2\pi R_1 h$ and $Q_2 = (2\sigma)\, 2\pi R_2 h$

0,25 0,25 0,25

$\Rightarrow Q_{int} = \sigma\, 2\pi h (R_1 + 2R_2)$ 0,5

So, $E_3\, 2\pi r h = \sigma\, 2\pi h (R_1 + 2R_2)$

$\Rightarrow E_3 = \dfrac{\sigma (R_1 + 2R_2)}{\varepsilon_0 r}$ 0,5

For $r = R_1$: $E_2(R_1) = \dfrac{\sigma}{\varepsilon_0}$ 0,25 and for $r = R_2$: $E(R_2) = \dfrac{\sigma(R_1 + 2R_2)}{\varepsilon_0 R_2}$ 0,25

2) The potential

$\vec{E} = -\text{grad}\, V$ 0,25

$\Rightarrow E = -\text{grad}\, V = -\dfrac{dV}{dr}$

$\Rightarrow dV = -E\, dr$

$\Rightarrow V = -\int E\, dr$ 0,25

• For $r \geqslant R_2$: $V_3 = -\int E_3\, dr = -\int \dfrac{\sigma(R_1 + 2R_2)}{\varepsilon_0 r}\, dr$

$= -\dfrac{\sigma(R_1 + 2R_2)}{\varepsilon_0} \int \dfrac{dr}{r}$

$= -\dfrac{\sigma(R_1 + 2R_2)}{\varepsilon_0} \ln r + C$ 0,5

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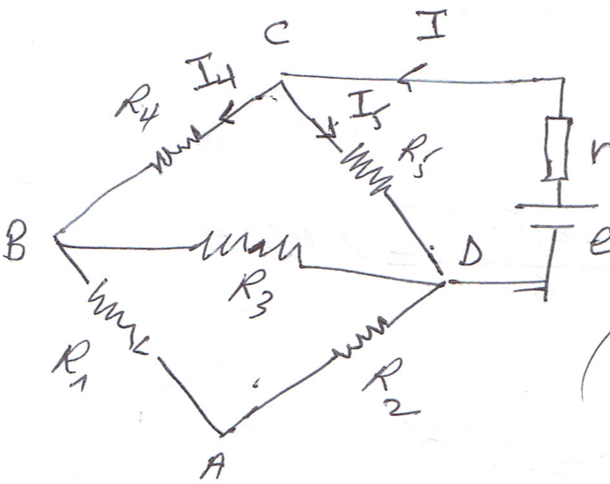
Ex 03

1/ The current intensity in each resistor



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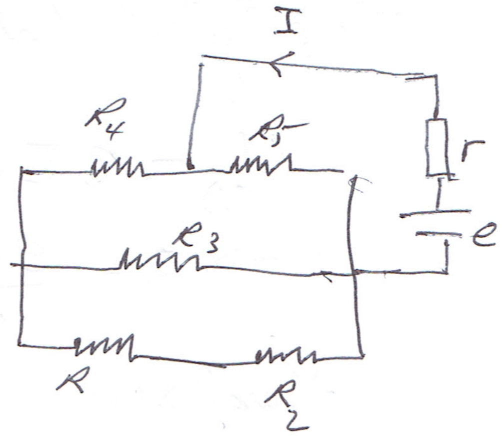
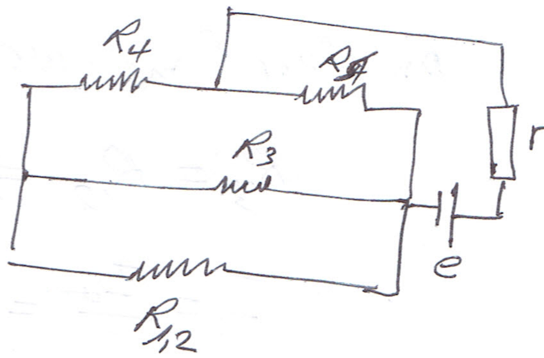


0,5

$$R_{1,2} = R_1 + R_2 = 1 + 1 = 2\,\Omega$$

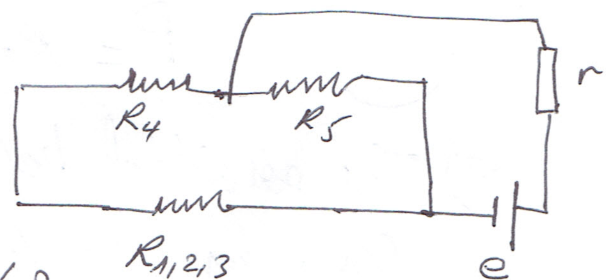
0,5



$$\frac{1}{R_{123}} = \frac{1}{R_{12}} + \frac{1}{R_3} \Rightarrow R_{1,3} = \frac{R_{12}\,R_3}{R_{12} + R_3}$$

0,5

$$\Rightarrow R_{123} = \frac{2 \times 2}{2+2} = 1\,\Omega$$



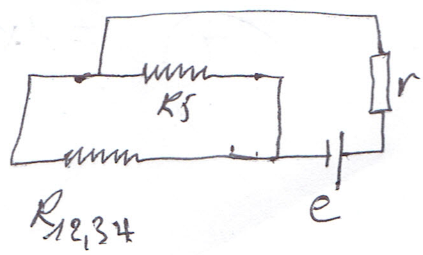
$$R_{1234} = R_{123} + R_4 = 1 + 1 = 2\,\Omega$$

0,5

$$\frac{1}{R_{12345}} = \frac{1}{R_{1234}} + \frac{1}{R_5} =$$

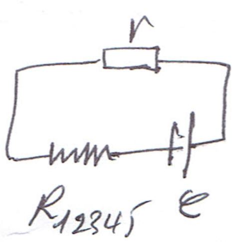
$$\Rightarrow R_{12345} = \frac{R_{1234} \times R_5}{R_{1234} + R_5} = \frac{2 \times 6}{2+6} = 1{,}5\,\Omega$$

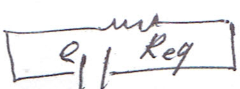
0,5



$$R_{eq} = R_{12345} + r = 1{,}5 + 0{,}5 = 2\,\Omega$$

0,5





The current I in the circuit is given by /5

$$I = \frac{e}{R_{eq}} = \frac{9}{2} = 4{,}5\,A$$ 0,5

The potential between C and D is

$$U_{CD} = R_5 I_5 = e - rI$$ 0,25

$$\Rightarrow I_5 = \frac{e - rI}{R_5} = \frac{9 - 0{,}5\times 4{,}5}{6} = 1{,}125\,A$$

therefore,

$$I_4 = I - I_5 = 4{,}5 - 1{,}125 = 3{,}375$$ 0,25

On other hand, we have

$$R_3 = R_{12} \Rightarrow I_3 = I_{12}$$ 0,25

So, $$I_3 = \frac{I_4}{2} = \frac{3{,}375}{2} = 1{,}687\,A$$

2) The power provided by the generator is

1 $$P = R_{eq} I^2 = eI = 9 \times 4{,}5 = 40{,}5\,W$$

3) The potential between A and C can be calculated from any branch, let be ADeC, so

$$U_{AC} = R_2 I_3 + rI - e$$

1 $$= 1\times 1{,}687 + 0{,}5\times 4{,}5 - 9 = -5\,V$$