

Mechanical Engineering Department

Subject: Strength of Materials

Level: L2GM

Duration: 1h30

## Solution to the final exam

### Course questions (05 pts)

1. The fundamental purpose of the study of Strength of Materials is: (1)
  - ✓ The objective of Strength of Materials is to determine whether a structure can withstand the applied loads and to verify that deformations remain within acceptable limits to ensure its proper functioning.
2. The difference between normal stress and shear stress is: (1)
  - ✓ Normal stress acts perpendicular to the cross-section (along the axis), while shear stress acts parallel to the cross-section (shearing).
3. The slope of the stress-strain curve represent in the elastic region is: (1)
  - ✓ It is the Young's modulus ( $E$ ), representing the stiffness of the material in elastic tension/compression: the higher  $E$  is, the stiffer the material.
4. The hypothesis is associated with the local validity of stresses away from load application areas is: (1)
  - ✓ This is Saint-Venant's hypothesis: the local effects of loads diminish quickly, and only the overall resultant forces and moments determine the stress state far from the load application zones.
5. Verification of shear and tension in a bolted assembly is: (1)
  - ✓ It is verified that:
    - The shear stress  $\tau$  does not exceed the allowable shear stress  $[\tau]$ ;
    - The normal stress  $\sigma$  in the bolt does not exceed the allowable normal stress  $[\sigma]$ ;
    - The hole dimensions are suitable to avoid play and ensure resistance against slipping.

ex 01 (67,50 pts)

1- The value of distributed axial load ( $q$ ) that ensures the bar is in static equilibrium:

$$\sum \vec{F}_{ext} = \vec{0}$$

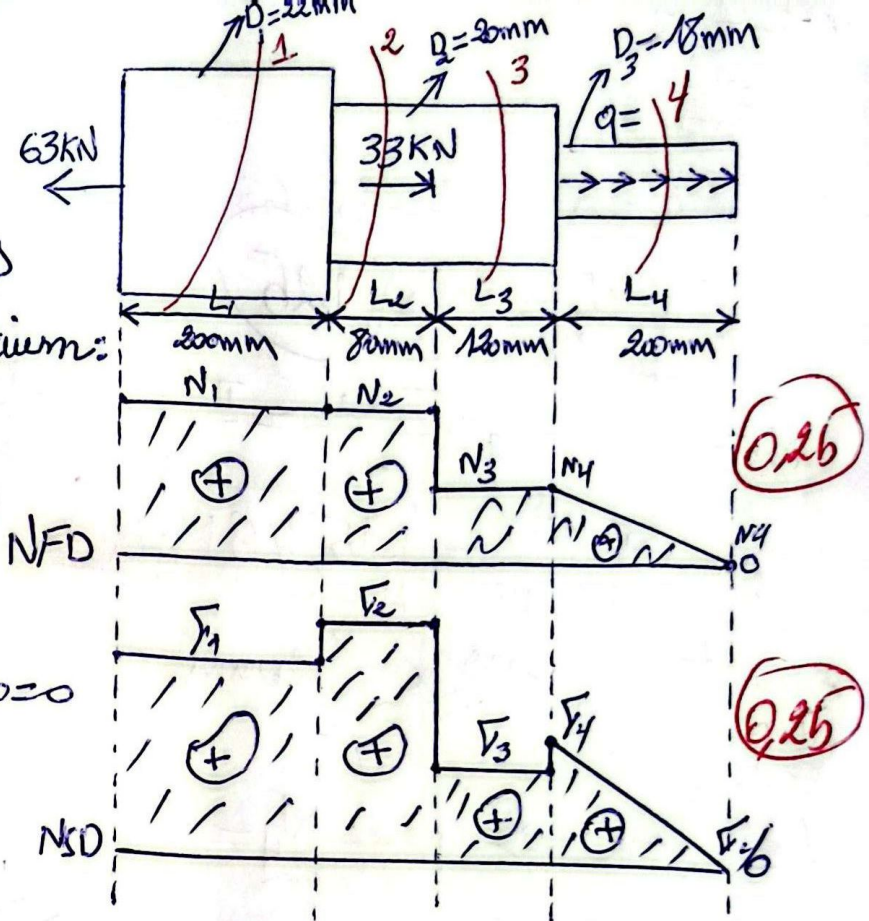
$$\sum F_x = 0$$

$$63 - 33 - q \times L_4 = 0$$

$$30 - q \times L_4 = 0 \Rightarrow 30 - q \times 200 = 0$$

$$\Rightarrow q = \frac{30}{200} = 0,15$$

$$|q = 0,15 \text{ kN/mm}|$$



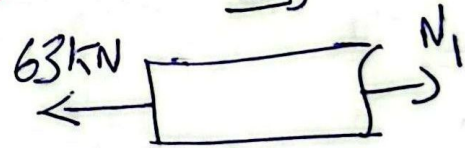
2- The normal force and normal stress diagram along the bar

\* Part 1:

$$\sum \vec{F}_{ext} = \vec{0} \Rightarrow N_1 - 63 = 0$$

$$\sum F_x = 0 \Rightarrow N_1 - 63 = 0$$

$$|N_1 = 63 \text{ kN}|$$



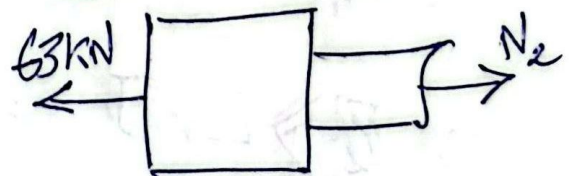
$$S_1 = \frac{N_1}{\sigma_1} = \frac{\pi d_1^2}{4} = \frac{\pi (22)^2}{4} = 380,13 \text{ mm}^2$$

$$\sigma_1 = \frac{N_1}{S_1} = \frac{63 \cdot 10^3}{380,13} = 165,73 \text{ MPa}$$

$$\sum \vec{F}_{ext} = \vec{0}$$

$$\sum F_x = 0 \Rightarrow N_2 - 63 = 0$$

$$|N_2 = 63 \text{ kN}|$$



$$S_2 = \frac{N_2}{\sigma_2} = \frac{\pi d_2^2}{4} = \frac{\pi (20)^2}{4} = 314,15 \text{ mm}^2$$

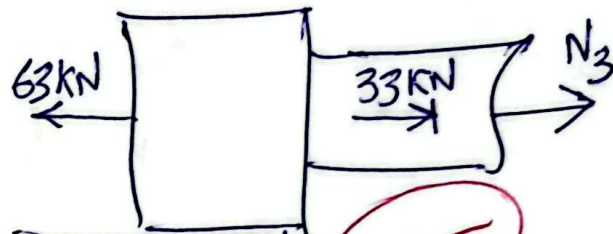
$$\sigma_2 = \frac{N_2}{S_2} = \frac{63 \cdot 10^3}{314,15} = 200,54 \text{ MPa}$$



$$\sum \vec{F}_{ext} = \vec{0}$$

$$x/ \sum F_x = 0 \Rightarrow N_3 + 33 - 63 = 0$$

$$N_3 = 30 \text{ kN} \quad (0,25)$$



$$S_3 = \frac{N_3}{\sigma_2} / S_2 = \frac{\pi d_2^2}{4} = \frac{\pi (20)^2}{4} = 314,15 \text{ mm}^2 \quad (0,25)$$

$$\sigma_3 = \frac{30 \cdot 10^3}{314,15} = 95,49 \text{ MPa} \quad (0,25)$$

\* Part 4  $0 \text{ mm} \leq x \leq 200 \text{ mm}$

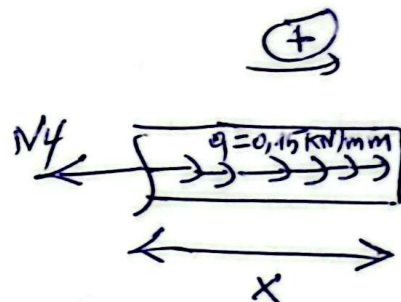
$$\sum \vec{F}_{ext} = \vec{0}$$

$$x/ \sum F_x = 0 \Rightarrow q \cdot x - N_4 = 0$$

$$N_4 = q \cdot x$$

$$\text{For } x = 0 \text{ mm} \Rightarrow N_4 = 0 \text{ kN} \quad (0,25)$$

$$\text{For } x = 200 \text{ mm} \Rightarrow N_4 = 0,15 \times 200 = 30 \text{ kN} \quad (0,25)$$



$$\text{For } x = 0 \text{ mm} \quad S_3 = \frac{N_4}{\sigma_3} / S_3 = \frac{\pi d_3^2}{4} = \frac{\pi (18)^2}{4} = 254,46 \text{ mm}^2 \quad (0,25)$$

$$\sigma_4 = \frac{0}{254,46} = 0 \text{ MPa} \quad (0,25)$$

For  $x = 200 \text{ mm}$

$$\sigma_4 = \frac{30 \cdot 10^3}{254,46} = 117,89 \text{ MPa} \quad (0,25)$$

3-Verification of the bar's strength

$$\sigma_2 = 200,54 \text{ MPa} > [\sigma] = 155 \text{ MPa} \quad (0,25)$$

The bar does not meet the strength criteria 0,25

4 - Calculate of the total deformation

$$\Delta L = \int_0^L \frac{N}{ES} dx = \int_0^{L_1} \frac{N_1}{ES_1} dx_1 + \int_{L_1}^{L_1+L_2} \frac{N_2}{ES_2} dx_2 + \int_{L_1+L_2}^{L_1+L_2+L_3} \frac{N_3}{ES_2} dx_3 + \int_{L_1+L_2+L_3}^{L_1+L_2+L_3+L_4} \frac{N_4}{ES_3} dx_4$$

0,25

$$\Delta L = \frac{N_1 L_1}{ES_1} + \frac{N_2 L_2}{ES_2} + \frac{N_3 L_3}{ES_2} + \frac{N_4 L_4}{ES_3}$$

$$\Delta L = \frac{1}{2 \times 10^8} \times \left( \frac{63 \times 10^3 \times 200}{380,13} + \frac{63 \times 10^3 \times 80}{314,15} + \frac{30 \times 10^3 \times 120}{314,15} + \frac{30 \times 10^3 \times 200}{254,46} \right)$$

$$\Delta L = \frac{1}{200} (33,14 + 16,04 + 11,45 + 23,57)$$

$$\Delta L = 0,42 \text{ mm}$$

1

5 Determination of the percentage change in length

$$P = \frac{\Delta L}{L_0} \times 100 \Rightarrow L_0 = L_1 + L_2 + L_3 + L_4 = 600 \text{ mm}$$

0,25

$$P = \frac{0,42}{600} \times 100 = 0,07\%$$

0,75



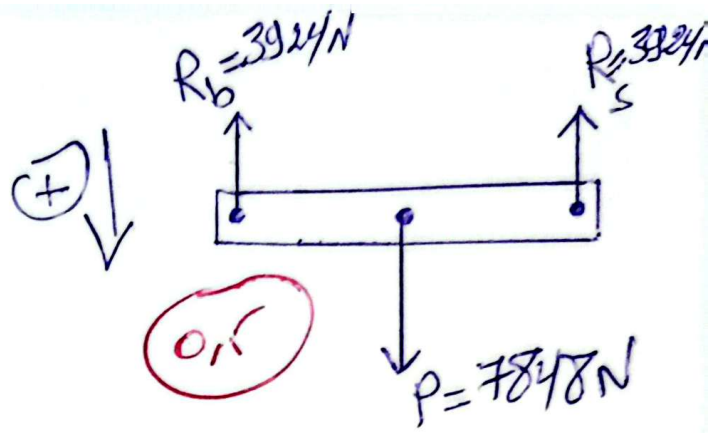
ex 02 (07,50 pts)

$$m = 800 \text{ kg}$$

$$g = 9,81 \text{ N/kg}$$

$$\sigma_b = 90 \text{ MPa}$$

$$\sigma_s = 120 \text{ MPa}$$



$$P = m \times g = 800 \times 9,81 = \boxed{7848 \text{ N}} \quad (1)$$

$$\sum \vec{F}_{ext} = \vec{0}$$

$$\forall \sum F_x = 0 \Rightarrow P - R_b - R_s = 0 \quad / \quad R = R_b = R_s$$

$$P - R - R = 0 \Rightarrow P - 2R = 0$$

$$R = \frac{P}{2} = \frac{7848}{2} = \boxed{3924 \text{ N}} \quad (1)$$

$$\sigma_b = \frac{R_b}{S_b} \Rightarrow 90 = \frac{3924}{S_b}$$

$$\Rightarrow S_b = \frac{3924}{90} = \boxed{43,6 \text{ mm}^2} \quad (2)$$

$$\sigma_s = \frac{R_s}{S_s} \Rightarrow 120 = \frac{3924}{S_s}$$

$$\Rightarrow S_s = \frac{3924}{120} = \boxed{32,7 \text{ mm}^2} \quad (2)$$