Final exam

Exercise01 **(05pts)**

Consider the function $f:[-1;1] \to [0;1]$ defined by $f(x) = \sin(\arccos(x))$

1- Calculate $f(0), f(1)$ *and* $f(-1)$.

2- Show that $f'(x) = -\frac{x}{\sqrt{1-x^2}}$ for $x \in]-1;1[$ then sketch the table of variation of the function f.

3- Show that

$$\forall x \in [-1;1] \sin(\arccos(x)) = \sqrt{1-x^2}$$

4- Deduce that the graph of f is the top half of unit circle.

Exercise02 **(08pts)**

1. Determine the reel constants a *and* b such that $\frac{9}{x^2-5x-14} = \frac{a}{x+2} + \frac{b}{x-7}$.
2. Find the indefinite integral $\int \frac{9}{x^2-5x-14}\, dx$.

   Deduce the value of definite integral $\int_0^1 \frac{9}{x^2-5x-14}\, dx$.
3. Use a suitable change of variable to evaluate $\int_0^{\frac{\pi}{2}} \frac{9\cos t}{-14-5\sin t+\sin^2 t}\, dt$.
4. Let $x \in]7;+\infty[$, solve the following first order differential equation:

$$y' - \frac{9}{x^2-5x-14} y = \frac{x-7}{x^2-5x-14}$$

Exercise02 **(07pts)**

Consider the following second order differential equation:

$$y'' - 4y' + 4y = (2x-4)e^x \; .... (2)$$

1. Solve the homogeneous differential equation given by: $y'' - 4y' + 4y = 0$.
2. Determine the constants α *and* β so that $y_{p=}(\alpha x + \beta)e^x$ is a particular solution of (2).
3. Find the general solution of (2).

Good luck.