

Correction of the final exam in: Fluids Mechanics

Exercise 1: 5 pts.

1) Calculation of:

Weight w ; Density ρ ; Specific gravity γ .

We have: $m = 60 \text{ kg}$, $V = 5 \text{ m}^3$.

$$\Rightarrow \rho = \frac{m}{V} = \frac{60}{5} = 12 \text{ kg} \cdot \text{m}^{-3} \quad (0.5)$$

$$\Rightarrow W = mg = 60 \times 9.81 = 588.6 \text{ N.} \quad (0.5)$$

$$\Rightarrow SG = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} = \frac{\rho \cdot g}{\rho_{\text{water}} \cdot g} = \frac{12 \times 9.81}{9810} \quad (0.5)$$

$$\{ SG = 0.012 \} \quad (0.5)$$

$$\text{or: } SG = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} = \frac{12}{1000} = 0.012.$$

2) Calculation of: Weight w :

$$\text{We have: } V = 2 \text{ l} = 2 \times 10^{-3} = 0.002 \text{ m}^3 \quad (0.5)$$

$$d = 0.918 = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} \Rightarrow \rho_{\text{liquid}} = 0.918 \times 1000$$

$$\{ \rho = 918 \text{ kg} \cdot \text{m}^{-3} \} \quad (0.5)$$

$$\text{So: } w = \rho \cdot V \cdot g = 918 \times 0.002 \times 9.81$$

$$\{ w = 18 \text{ N} \} \quad (0.5)$$

Exercise 2: 7 pts.

Calculation of: Pressure at point A

$$P_A = ?$$

To calculate P_A we apply the fundamental equation of statics between points: (0.5)
A-B; A-C; C-D and D-E.

$$(0.5) \rightarrow P_A - P_B = \rho_w \cdot g (h_B - h_A) \quad (0.5) \rightarrow P_B = P_C = P_{\text{air}}$$

$$(0.5) \rightarrow P_C - P_D = \rho_{Hg} \cdot g (h_D - h_C) \quad (0.5) \rightarrow P_D - P_E = \rho_{\text{oil}} \cdot g (h_E - h_D)$$

$$\text{So: } P_A - P_E = P_A - P_B + P_C - P_D + P_D - P_E = P_A - P_E.$$

$$\rightarrow P_A - P_E = \rho_w g (h_B - h_A) + \rho_{Hg} (h_D - h_C) + \rho_{\text{oil}} g (h_E - h_D) \quad (1)$$

$$\rightarrow P_A - P_E = \rho_w g (0,65) + \rho_{Hg} g (-0,25) + \rho_{\text{oil}} \cdot g (0,60) \quad (1)$$

$$(0.5) \text{ we know that: } d = \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}} \Rightarrow \rho_{\text{fluid}} = d \cdot \rho_{\text{water}}$$

So:

$$(0.5) \quad P_A - P_E = g \cdot \rho_{\text{water}} (d (0,65) - d_{Hg} (0,25) + d_{\text{oil}} (0,6)) \quad (1)$$

$$(0.5) \quad P_E = P_{\text{atm}} = 10^5 \text{ Pa}$$

$$\text{So: } P_A = P_{\text{at}} + (9,81 \times 1000 \times [0,65 - 13,6(0,25) + 0,85(0,6)])$$

$$P_A = 78025,6 \text{ Pa} \quad (1)$$

Exercise 03: 8 pts.

1) Calculation of P_B at B:

$$P_B - P_{B'} = \rho g (h_{B'} - h_B) \quad (0.5), \quad P_{B'} = 1.01 \cdot 10^5 \text{ Pa.}$$

$$\Rightarrow P_B = P_{B'} + \rho g (h_{B'} - h_B) \quad (0.25)$$

$$\text{so: } P_B = 1.01 \cdot 10^5 + 1000 \cdot 9.81 (3 - 0)$$

$$\boxed{P_B = 1.3 \times 10^5 \text{ Pa}} \quad (0.5)$$

2) Calculation of P_A at A:

$$P_A - P_{A'} = \rho g (h_{A'} - h_A) \quad (0.5)$$

$$P_A = P_{A'} + \rho g (h_{A'} - h_A) \quad (0.25)$$

$$\text{so: } P_A = 1.01 \cdot 10^5 + 1000 \cdot 9.81 (6 - 0)$$

$$\Rightarrow \boxed{P_A = 1.6 \times 10^5 \text{ Pa}} \quad (0.5)$$

3) The continuity equation between

$$A \text{ and } B: S_A \cdot v_A = S_B \cdot v_B \quad (0.5)$$

$$\Rightarrow v_B = \frac{S_A}{S_B} v_A = \frac{D_A^2}{D_B^2} v_A \Rightarrow \boxed{v_B = 4 v_A} \quad (0.5)$$

4) Bernoulli's equation between A and B:

$$P_A + \frac{1}{2} \rho v_A^2 + \rho g h_A = P_B + \frac{1}{2} \rho v_B^2 + \rho g h_B \quad (1)$$

* The flow velocity v_B :

$$h_A = h_B \Rightarrow P_A + \rho \frac{v_B^2 \cdot 1}{16} = P_B + \frac{1}{2} \rho v_B^2 \quad (0,5)$$

$$v_A = \frac{v_B}{4} \Rightarrow \frac{1}{2} \rho \left(\frac{v_B^2}{16} - v_B^2 \right) = P_B - P_A$$

$$\Rightarrow v_B = \sqrt{\frac{-32(P_B - P_A)}{15 \cdot \rho}} = \sqrt{\frac{-32(1,3 - 1,6) \cdot 10^5}{15 \times 1000}} \quad (0,5)$$

$$v_B = 8 \text{ m/s} \quad (0,5)$$

5) The flow rate Q_v :

$$Q_v = S_A v_A = S_B v_B \quad (0,5)$$

$$S_B = \pi \frac{D_B^2}{4} = \frac{3,14 \times (20 \times 10^{-2})^2}{4} = 0,0314 \text{ m}^2 \quad (0,5)$$

$$\text{So: } Q_v = 0,0314 \times 8 = 0,25 \text{ m}^3/\text{s} \quad (0,5)$$