People's Democratic Republic of Algeria

Ministry of Higher Education and Scientific Research

University of Abbas Laghrour – Khenchela

Faculty of Economic Sciences, Commerce, and Management Sciences

Department of Management Sciences

Subject: Statistics03

Field/Specialization: Management

Date: 14/01/2026

Time: 11:00 – 12:30

Model Answer for the Exam in the Subject: Statistics03.

Ordinary Session

Academic Year: 2025–2026

Course Instructor: Mouhcene Hamrit

Model Answer with Marking Scheme

Exo:01

If events occur according to a Poisson process with average rate λ per unit time, then the number of events X in a time interval of length t follows a Poisson distribution with mean parameter λt

The probability mass function

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

1. Probability that exactly 5 customers arrive in the next hour

$P(X = 5) = \frac{8^5}{5!} e^{-8} \approx 0.0916$……………………………………**1.5 points**

2. **Probability that at most 3 customers arrive in the next hour**

$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) =$
$e^{-8} [\frac{8^0}{0!} + \frac{8}{1!} + \frac{8^2}{2!} + \frac{8^3}{3!}] \cong 0.0424$…………………..………..**1.5 points**

3. **Probability that no customers arrive in the next 30 minutes**

$$\lambda t = 8 * \frac{30}{60} = 4$$

$P(X = 0) = \frac{4^0}{0!} e^{-4} \approx 0.0183$……………………………..…**1.5 points**

4. **Mean and variance of number of customers arriving in 90 minutes**

$E(X) = Var(X) = \lambda t = 8 * \frac{90}{60} = 12$………………………………..…..**2 points**

Exo 02:

All four conditions are satisfied to apply the geometric distribution:

- Binary outcomes
- Independent trials
- Constant defect probability *p*=0.08
- Counting trials up to and including the first defective

The probability mass function is : $P(X = x) = (1 - p)^{x-1} * p$

1. $P(X = 3)$ *first two components are **good**, third is **defective***
   $P(X = 3) = (1 - p)^2 p = 0.92^2 * 0.08 \cong 0.067$………………..……***1.5 points***

2. $P(X > 5) = (1 - p)^4 = 0.92^4 \cong 0.7164$………………………………..***1.5 points***

3. $E(X) = \frac{1}{p} = \frac{1}{0.08} = 12.5,\ \ var(X) = \frac{1-p}{p^\wedge 2} = \frac{1-0.08}{0.08^2} \cong 143.75$……………***1.5points***

$p(X \geq 8/X > 4) = \frac{P(X\geq 8 \cap X>4)}{P(X>4)} = \frac{P(X\geq 8)}{P(X>4} = \frac{(1-p)^7}{(1-p)^4} = (1-p)^3 = P(X > 3) \cong 0.7787$……………..***1 point***

$P(X \geq 8) = P(X > 7) = (1-p)^7$

The memoryless property of the geometric distribution states:

For any $n, m \ \geq 1$

$$P(X > n + m/X > m) = P(X > n)$$

In words: **The probability of waiting at least *n* more trials for the first success, given you've already waited *m* trials without success, is the same as if you were starting fresh**

We've already tested 4 components and seen no defect → X>4, We want the probability that we need to wait at least 8 total, i.e., at least 4 more trials beyond trial 4 → that's X≥8, or equivalently X>7

Even though we've already tested 4 good components, the process "forgets" that history. The chance that we'll need at least 3 more good components before a defect is exactly the same as it was at the very beginning…………………………**1 point**

Exo 03:

1. Probability the fuse lasts at least 1 year

$$P(X \geq x) = 1 - F(x) = e^{-\lambda x}$$

$P(X \geq 1) = P(X > 1) = e^{-\lambda} = e^{-2} \cong 0.1353$……………………**1 point**

2. **Probability the fuse fails within the first 0.5 years**

$$P(X \leq x) = 1 - e^{-\lambda x}$$

$P(X \leq 0.5) = 1 - e^{-2*0.5} = 1 - e^{-1} \cong 0.6321$……………………**1 point**

2. Median lifetime of the fuse

$$F(m) = \frac{1}{2} \Rightarrow 1 - e^{-\lambda m} = \frac{1}{2}$$

$e^{2m} = \frac{1}{2} \Rightarrow m = 0.3466\ year$…………………………..**1 point**

Exo:04

1. The statistic T follows a Student's t-distribution with:

Degrees of freedom=n−1=21−1=20 ………………………..**1 point**

2. Compute $P(T > 2.845)$

We need the upper-tail probability for a t_{20} distribution at t=2.845

from standard t-tables: For df=20 , the critical value for a one-tailed probability of 0.005 is:

$t_{0.005,20} = 2.845$ so $P(T > 2.845) = 0.005$………………**1 point**

3. **Determine the symmetric interval** (−t∗,t∗) **such that** P(−t∗<T<t∗)=0.99

   This is the **central 99% interval** for a t_{20} distribution., That leaves **1% in the tails**, i.e., **0.5% in each tail**. So, we need the **0.995 quantile** $P(T < t^*) = 0.995$ *or* $P(T > t^*) = 0.005$. From t-table:
   $t_{0.005,20} = 2.845$ since t distribution is symmetric $P(T < -2.845) = 0.005$
   Thus:
   $P(-2.845 < T < 2.845) = 0.99$…………………………**1 point**

4. **Find the value** *t* **such** $P(T < t) = 0.95$

This is the 95th percentile (or 0.05 upper-tail) of the t_{20} distribution.

From standard t-tables:

$$P(T < t) = 1 - P(T > t) \Rightarrow P(T > t) = 1 - P(T < t) = 1 - 0.95 = 0.05$$

At t=20 degrees of freedom row and $\alpha = 0.05$ we get t = 1.725…………………**1 point**